MATH 347: FUNDAMENTAL MATHEMATICS, FALL 2015

HOMEWORK 4

Due on Wednesday, Sep 23

Exercises from the textbook. 3.2–3.4, 3.11, 3.14(a), 3.16, 3.17, 3.22, 3.28[1], 3.31[2].

Out-of-the-textbook exercises (these are as mandatory as the ones from the textbook)**.**

1. For a subset $A \subseteq \mathbb{N}$, a number $n \in \mathbb{N}$ is called *a least element of* A if $n \in A$ and for all $m \in A$, $n \leq m$.

(a) (Uniqueness) Show that the least element is unique, i.e. every subset $A \subseteq \mathbb{N}$ can have at most one least element.

HINT: The statement you want to prove is this:

$$\forall n, m \in \mathbb{N} \left[n \text{ and } m \text{ are least elements of } A \Rightarrow n = m \right].$$

Prove this directly.

(b) (Existence) Using the method of induction to prove that every nonempty subset of $\mathbb{N}$ has a least element.

HINT: The statement you want to prove is this:

$$\forall A \subseteq \mathbb{N}, A \neq \varnothing \Rightarrow A \text{ has a least element.}$$

Prove the contrapositive of the implication, i.e. suppose that A does not have a least element and show that $A = \varnothing$.

2. In class, we proved the Induction Principle (Theorem 3.6) using the definition of $\mathbb{N}$ (Definition 3.5). Give an alternative proof of Theorem 3.6 using the fact (stated above in Problem 1.(b)) that every nonempty subset of $\mathbb{N}$ has a least element.

[1] HINT FOR 3.28: To find the formula, write $\frac{1}{i(i+1)}$ in terms of $\frac{1}{i}$ and $\frac{1}{i+1}$, expand the $\sum$ (write the first and last three terms), and notice that many things cancel.

[2] HINT FOR 3.31: To find the formula, write $1 - \frac{1}{i^2}$ as $\frac{(i-1)(i+1)}{i^2}$, expand the $\prod$ (write the first and last three terms), and notice that many things cancel.